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A COMPARISON OF TWO MASS DISTRIBUTIONS APPLICABLE TO GLOBULAR CLUSTERS AND DWARF GALAXIES

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SUMMARY: A particular case of mass distribution in stellar systems, already described in the literature, is compared to the King model of mass distribution. For the cases which would correspond to the description of real stellar systems, such as the globular clusters and dwarf galaxies, one finds a satisfactory agreement between these two mass distributions.

Key words. globular clusters: general – Galaxy: halo – Galaxies: dwarf

1. INTRODUCTION

It is well known that as a result of his studies of globular clusters King (1962) proposed an empirical formula for mass distribution. These studies were extended afterwards to include a more general treatment of star distribution within a globular cluster, or a similar stellar system (e.g. King 1966). On the other hand, it was noticed by Veltmann (1961) that the density formula obtained by King (1962) had a similarity with another form of mass distribution, usually referred to as the generalized Schuster density law (here a particular value of the exponent in the formula is borne in mind).

The difficulties with King's mass distribution are rather well known; for instance, from its density formula no analytical solutions for the cumulative mass and potential can be obtained. This circumstance was borne in mind in a recent paper of the present author (Ninković 2003 - herein referred to as Paper I) when some improvements were suggested. However, both King's original formula and Ninković's modifications concern the case where the volume density reaches zero at a finite distance to the centre of the system. As already said in Paper I, it could seem more natural if the density at the limiting radius were still non-zero. For this reason the possibility of applying a mass distribution with a discontinuity at the boundary of the system is considered in the present paper.

2. THE CASE UNDER STUDY

This is a particular case of the generalized Schuster density law - $i = 3$ (e. g. Ninković 1998). The density formula is

$$\rho(r) = \frac{\rho(0)}{[1 + (r/r_\mathrm{c})^2]^{3/2}} \; ; \qquad (1)$$

ρ is the density, r is the distance to the centre, whereas r_c is a constant (usually called core radius). Eq. (1) may be applied within a finite distance $r = r_\mathrm{l}$ only, because otherwise it would yield an infinite total mass. As a consequence the density at $r = r_\mathrm{l}-$ will be non-zero, whereas at $r = r_\mathrm{l}+$ it will be zero. In addition, on the basis of Eq. (1) the corresponding formula for the surface density can be easily obtained (e.g. Ninković 1998)

$$\sigma(\tilde{r}) = \frac{2\rho(0) r_\mathrm{c}^3}{(r_\mathrm{l}^2 + r_\mathrm{c}^2)^{1/2}} \frac{(r_\mathrm{l}^2 - \tilde{r}^2)^{1/2}}{r_\mathrm{c}^2 + \tilde{r}^2} \; ,$$

unlike the case studied in Paper I where no analytical form for the surface density can be obtained; $\tilde{r}$ is the distance to the centre in the tangential plane.

The problem of defining the outer boundary of a stellar system is rather well known (e.g. Ogorodnikov 1958 - p. 483). The fact that the total mechanical energy for every star in a gravitationally bound stellar system must be negative is of no use because it only indicates that the apocentric distances of all stars are finite, but yields no particular limit for any of them. In other words, by using this condition only it is impossible to determine the maximal apocentric distance in the system. It is quite clear that the maximal apocentric distance should be equal to the limiting radius of the system $r_{\rm l}$. However, any observational estimate of the limiting radius is very difficult. This is the reason why this quantity is not defined if the cumulative mass of a stellar system is convergent. However, if the cumulative mass is divergent, as in the case of (1), then the limiting radius must be defined. Due to the difficulties following the determination of the limiting radius, mentioned above, it cannot be expected that all stars of a stellar system are surely within an observationally estimated limiting radius. In brief, existing estimates of limiting radii in stellar systems are rather crude and, consequently, they should be aimed at comprising a sufficiently high percentage of system stars so that the gravitation field beyond is described well enough with the point-mass formula. Therefore, two different models of mass distribution, when fitted to the observations, need not always yield the same limiting radius. It is more important that the total masses are approximately equal.

As models of mass distribution to be compared in the present paper King's (1962) one and that represented by Eq. (1) are chosen. King's formula for the volume density is

$$\rho(r) = \frac{\rho(0)}{y} \frac{1}{y^2 \arccos y^{-1} - x_{\rm t}} \cdot \\ \cdot \frac{1}{X^2} [X^{-1} \arccos X - (1-X^2)^{1/2}] \; ; \qquad (2)$$

$y = (1+x_{\rm t}^2)^{1/2}$, $X = \frac{(1+x^2)^{1/2}}{y}$, $x = r/r_{\rm c}, x_{\rm t} = r_{\rm t}/r_{\rm c}$

$r_{\rm t}$ is the limiting radius (in King's paper from 1962 named tidal radius). The corresponding surface density is

$$\sigma(\tilde{r}) = \frac{\pi r_{\rm c} \rho(0)}{y^2 \arccos y^{-1} - x_{\rm t}} \; [X^{-1} - 1]^2.$$

In the surface-density formula X is a function of $\tilde{r}$ with the same meaning as in the case of the volume-density formula.

3. THE COMPARISON

Since both density Eqs. ((1) and (2)) contain as parameters the central density and the core radius, in the present comparison the same values will be assumed for these two quantities in both models. For this reason, as more natural, the central density and the core radius are assumed as the units of density and distance to the centre, respectively. Consequently, the product $\rho(0) r_{\rm c}^3$ will appear as the mass unit. In both cases the total mass will also depend on the ratio $r_{\rm l}/r_{\rm c}$.

The present comparison can be done in two phases: density comparison and that of the total mass. As for the former one, the density in the case of King's formula, in addition to the ratio $r/r_{\rm c}$, also depends on $r_{\rm l}/r_{\rm c}$ as a parameter, whereas in that of Eq. (1) the density, as easily seen, depends on the variable $r/r_{\rm c}$ only. This affects the density comparison, but not too much (Fig. 1).

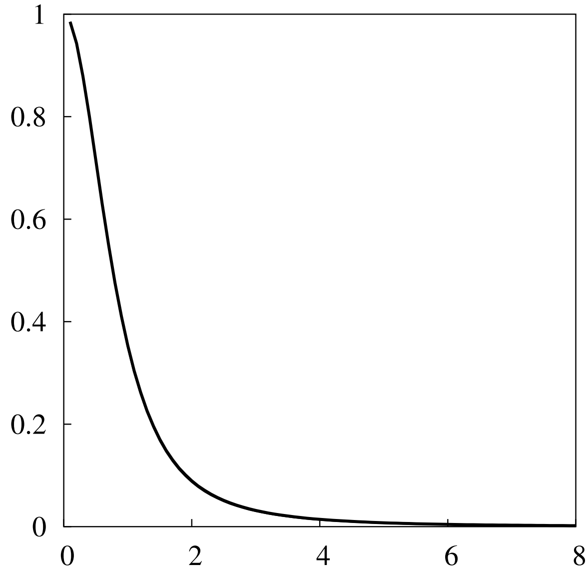

Fig. 1. *Density dependence: $y = \rho/\rho(0)$ is dimensionless density, $x = r/r_{\rm c}$ is dimensionless radius, the solid curve corresponds to formula (1), the dashed curve corresponds to formula (2), $r_{\rm l}/r_{\rm c} = 60$ for both, the central densities and radii $r_{\rm c}$ are equal in both models. The two curves practically coincide.*

It should be said that with regard to the possibilities of the density determination in real stellar systems there is no need to insist on a very close agreement between two model density functions. Therefore, a better approach appears to be to examine the total masses. Of course, in this case one assumes that the central density and the core radius are the same for both distributions so that the mass unit is already defined.

The first question to be answered then concerns, of course, the ratio of the total masses yielded by the two models examined here, where the ratio $r_{\rm l}/r_{\rm c}$ appears as the parameter. The calculation shows that for the case of the same $r_{\rm l}/r_{\rm c}$ mass distribution (1) always yields a higher total mass than King's one. As $r_{\rm l}/r_{\rm c}$ increases, the ratio of the total masses decreases, to reach the value of 1 for $r_{\rm l}/r_{\rm c} \to \infty$. The results are presented in Table 1.

Table 1. The ratio of total masses for two mass distributions as function of $r_\mathrm{l}/r_\mathrm{c} = x_\mathrm{t}$.

x_t	total-mass ratio
10	1.75
20	1.58
30	1.51
40	1.46
50	1.43
60	1.41
70	1.39
80	1.38
90	1.37
100	1.36

Table 2. Comparison of the two mass distributions.

x_t	$\mathcal{M}_\mathrm{k}$	x_crit	$x_\mathrm{crit}/x_\mathrm{t}$	$\rho(x_\mathrm{crit})$	f
10	14.41	4.10	0.41	7.3×10^{-3}	23%
20	21.41	7.38	0.37	1.4×10^{-3}	21%
30	25.82	10.54	0.35	4.9×10^{-4}	19%
40	29.06	13.60	0.34	2.3×10^{-4}	17%
50	31.62	16.80	0.336	1.3×10^{-4}	15%
60	33.74	19.80	0.330	7.7×10^{-5}	15%
70	35.53	23.00	0.329	4.9×10^{-5}	14%
80	37.12	26.00	0.325	3.4×10^{-5}	14%
90	38.52	29.10	0.323	2.5×10^{-5}	13%
100	39.78	32.20	0.322	1.8×10^{-5}	13%

However, as already said in Section 2, it is not necessary to have the same limiting radius, the equality rather concerns the total masses. Therefore, the present examination is continued by imposing this condition. Now one looks for the distance to the centre within which in the model characterised by formula (1) a mass equal to the total mass in the case of King's model is contained. The results are presented in Table 2. As the distance unit, as already said, r_c is used. In order to make the things more clear the designation for the ratio of the limiting radius to r_c in the case of King's model is x_t, whereas the one for the radius at which the cumulative mass corresponding to formula (1) is equal to the total mass yielded by King's model (unit r_c) is x_crit. The second column presents the total mass for King's model $\mathcal{M}_\mathrm{k}$ (as already said, in $\rho(0)r_\mathrm{c}^3$), the fourth one the ratio $x_\mathrm{crit}/x_\mathrm{t}$; as for the remaining two columns, they contain the density (in units $\rho(0)$) corresponding to x_crit for King's model and the fraction of the total mass contained beyond x_crit for the same model, respectively.

4. DISCUSSION AND CONCLUSIONS

The main results of the present paper are given in Table 2. As seen from the table, the ratio $x_\mathrm{crit}/x_\mathrm{t}$ is a slowly decreasing function of x_t. With regard to Table 1 this could seem surprising, but an explanation can be immediately found after inspecting the fifth and sixth columns of Table 2. The density in King's model for x_crit decreases rather strongly with x_t increasing so that, consequently, the fraction of the total mass contained between x_crit and x_t in King's model is a decreasing function of x_t. It is clearly seen that for x_t high enough, say 80-100, this fraction is much smaller than for low x_t (say, 10-20). Thus at high x_t the density values corresponding to x_crit become very low and the fraction of the total mass contained in these very external parts of a stellar system tends to become negligible.

In principle, when such mass distributions, like the two examined here, are applied for the purpose of describing real stellar systems, one cannot expect the ratio $r_\mathrm{l}/r_\mathrm{c}$ to be about 1, especially for dwarf galaxies and globular clusters are borne in mind where this ratio is usually of the order of 10^1, even 10^2. In the present discussion the fact that the tolerance in the determination of the total mass for stellar systems is rather ample, say 20%, or even more, should be taken into account. In other words, a value of the total mass for given $\rho(0)$ and r_c resulting from the King model should not be treated as "absolutely correct". Therefore, one may conclude that the agreement of the two models of mass distribution considered here is quite satisfactory and in view of this the model based on formula (1) can be used in the treatment of real stellar systems. This could be a task for the future.

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ПОРЕЂЕЊЕ ДВАЈУ РАСПОДЕЛА МАСЕ ПРИМЕНЉИВИХ НА ЗБИЈЕНА ЗВЕЗДАНА ЈАТА И ПАТУЉАСТЕ ГАЛАКСИЈЕ

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Један конкретан случај расподеле масе у звезданим системима, већ описиван у литератури, пореди се са Кинговим моделом расподеле масе. Између ове две расподеле нађено је задовољавајуће слагање за случајеве који би одговарали опису стварних звезданих система као што су збијена јата и патуљасте галаксије.